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Free Vibration Identification of the Geometrically Nonlinear Isolator with Elastic Rings by using Hilbert Transform

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ABSTRACT

The geometrically nonlinear isolator formed by a pair of elastic circular springs in the push-pull configuration has the symmetrical hardening stiffness under static compression and tension. Thus, it could be a potential solution to satisfy the dual isolation requirements of steady-state vibrations and transient shocks in the engineering application. The nonlinear transmissibility of this isolator under large-amplitude sinusoidal excitations has been investigated theoretically and experimentally in our previous research. In this paper, the Hilbert transform is applied to identify the geometrically nonlinear isolator with measured free vibration responses in the time domain. The measured responses are acquired by a laser vibrometer with large initial deformations. Since all the involved instantaneous modal parameters contain fast oscillations around their average values, the empirical mode decomposition is employed to smooth the identified results of the instantaneous frequency and damping coefficient. It is found that the backbone curve obtained experimentally conforms well to the previously measured frequency responses. The identified nonlinear stiffness and damping force characteristics of this geometrically nonlinear isolator have good agreements with the results from the theoretically calculation and the frequency-domain test in our previous research. Therefore, this research provides an efficient approach to analyze the dynamic characteristics of the geometrically nonlinear isolator with push-pull configuration rings and is also beneficial to design the parameters of this isolator.

Keywords: system identification, free vibration, geometrical nonlinearity, Hilbert transform, instantaneous modal frequency

1. Introduction

In engineering application, small-amplitude steady-state vibrations and large-amplitude transient shocks often should be isolated simultaneously. Isolators with unique characteristics of the nonlinear stiffness or damping could satisfy these requirements. Thus, studies on analysis and design of nonlinear isolators become greatly significant in engineering practice.

Researches on a type of geometrically nonlinear isolators composed by circular rings were summarized by Ibrahim [1]. For composite circular spring, static characteristics under the uniaxial compression was studied by Tse [2] and a softening performance was found in its compression region, while it performed like a hardening spring under the uniaxial tension[3]. Furthermore, a pair of circular springs in the push-pull configuration was proposed in Ref. [4]. With the symmetrical hardening stiffness under static compression and tension, the push-pull configuration elastic rings would be a potential solution for isolating vibrations and shocks. In our previous research [5], an analytic method combined the geometrically nonlinear theory of curved beams and the harmonic balance method was proposed to study the dynamic properties of this type isolator. This method overcomes the difficulty in calculating the vibration with large deformations and succeeds in investigating the nonlinear dynamic characteristics of lock situation and usual jump. However, the mathematical modeling, numerical simulations and verified experiments were totally based on the steady sinusoidal excitation in the frequency domain. Therefore, study on the transient response of this geometrically nonlinear isolator should find another effective way.

Nowadays, Hilbert transform becomes a powerful analytical approach to non-stationary and nonlinear vibration in the time

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domain. Feldman [6] summarized a tutorial review on Hilbert transform methodology and its application of nonlinear mechanical system identification based on the previous researches [7-8]. It reveals an important problem that all modal parameters contain slowly varying primary solutions and fast-oscillated variations [6,9]. The traditional lowpass filtering or averaging simply cuts down the fast oscillations and ignores high order super-harmonics caused by the nonlinearity. Thus, a new technique called Hilbert vibration decomposition (HVD) was proposed to decompose time-varying vibration signals [8]. Besides, an extremely powerful and popular method called empirical mode decomposition (EMD) first introduced by Huang et al.[10] can adaptively decompose a non-stationary signal into a series of intrinsic mode functions (IMF). The algorithm of EMD was presented in Ref. [11], in which the cubic spline interpolating is applied to local maxima and minima to extract IMFs.

In this paper, the Hilbert transform is applied to identify the geometrically nonlinear isolator with measured data in free-vibration. This paper is organized as follows: Section 2 introduces the theoretical foundation of the geometrically nonlinear isolator and the Hilbert transform for identifying dynamic system. Section 3 shows the experimental setup and gives specific identified results, comparisons and discussions. Section4 is the conclusion of this paper.

2. Theoretical foundation

2.1 Geometrically nonlinear isolator

The isolator to be identified in this paper consists of two groups of push-pull configuration rings shown in Fig.1(a). One side of each ring is fixed on the fixed base and the other side is connected to the mass block rigidly. The mass block is restrained by a slider to make it as a single degree-of-freedom (SDOF) system. To ignore the gravity's effect, the whole system is put in the horizontal plane, which will simplify the problem as a symmetric nonlinearity. The displacement response of the mass is denoted by $x(t)$. Therefore, this isolator is equal to a general analytical model with nonlinear stiffness and damping shown in Fig.1(b).

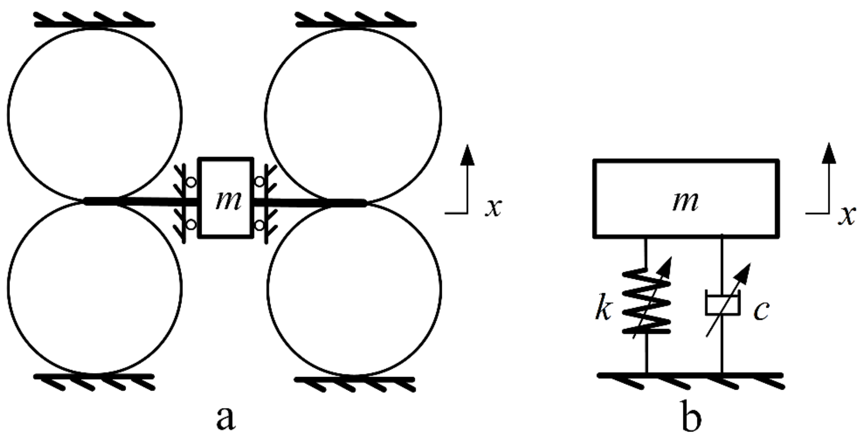

Fig. 1 The isolator with push-pull configuration rings: (a). real model; (b). equivalent model

The dynamic equation of this isolator in free vibration can be expressed

$$m\ddot{x}+F_c\left(\dot{x}\right)+F_k\left(x\right)=0 \tag{1}$$

where $F_k\left(x\right)$ is the nonlinear restoring force generated by deformed rings and $F_c\left(\dot{x}\right)$ is the nonlinear damping force which could include viscous, coulomb and other more complicated damping. A non-dimensional transformation with respect to the mass is applied to make the procedure of identification more convenient. The non-dimensional format of Eq.(1) can be formulated

$$\ddot{x}+f_c\left(\dot{x}\right)+f_k\left(x\right)=0 \tag{2}$$

According to approximate analytical methods for Eq.(2), $f_k\left(x\right)$, $f_c\left(\dot{x}\right)$ can often be expressed as

$$f_k\left(x\right)=k\left(x\right)x,\ \ c_k\left(x\right)=c\left(\dot{x}\right)\dot{x} \tag{3}$$

where $k\left(x\right),c\left(\dot{x}\right)$ denote equivalent stiffness and damping in some sense, such as the balance of harmonic components in frequency domain. For the nonstationary process in free vibration, the nonlinear restoring and damping force can be transformed into a multiplication form $k\left(x\right)x=k\left(t\right)x\left(t\right)$ and $c\left(\dot{x}\right)\dot{x}=c\left(t\right)\dot{x}\left(t\right)$, respectively. As a result, the dynamic equation of the isolator in free vibration is obtained as

$$\ddot{x}(t)+c(t)\dot{x}(t)+k(t)x(t)=0 \tag{4}$$

where *k(t)* and *c(t)* denote the varying instantaneous stiffness and damping.

2.2 Identification with Hilbert transform

By using the Hilbert transform in Eq.(4), the following equation can be obtained as

$$\ddot{X}(t)+c(t)\dot{X}(t)+k(t)X(t)=0 \tag{5}$$

where $X(t)$ is the analytic signal of $x(t)$ and $X(t)=x(t)+i\tilde{x}(t)=A(t)e^{i\psi(t)}$, in which $\tilde{x}(t)$ denotes the Hilbert transform, defined by

$$\tilde{x}(t)=H[x(t)]=1/\pi\int_{-\infty}^{\infty}x(\tau)/(t-\tau)d\tau \tag{6}$$

According to the analytic signal theory, the instantaneous displacement envelope and phase can be determined by

$$A(t)=\sqrt{x^2(t)+\tilde{x}^2(t)},\ \psi(t)=\arctan\left(\tilde{x}(t)/x(t)\right) \tag{7}$$

Thus, the first two order derivatives of $A(t)$ and $\psi(t)$ can be directly derived from Eq.(7) with measured $x(t)$ and its HT $\tilde{x}(t)$. The first two order derivatives of $X(t)$ can be formulated as

$$\begin{aligned}\dot{X}(t)&=X(t)\left[\dot{A}(t)/A(t)+i\dot{\psi}(t)\right]\\ \ddot{X}(t)&=X(t)\left[\ddot{A}(t)/A(t)+2i\dot{\psi}(t)\dot{A}(t)/A(t)-\dot{\psi}^2(t)+i\ddot{\psi}(t)\right]\end{aligned} \tag{8}$$

Substituting Eq.(8) into Eq.(5) and equating the real, imagine parts on both sides of Eq.(5) yield two equations

$$\begin{aligned}k(t)&=\dot{\psi}^2(t)-\dot{A}(t)/A(t)+2\dot{A}^2(t)/A^2(t)+\dot{A}(t)\ddot{\psi}(t)/\left(A(t)\dot{\psi}(t)\right)\\ c(t)&=-2\dot{A}(t)/A(t)-\ddot{\psi}(t)/\dot{\psi}(t)\end{aligned} \tag{9}$$

According to the first two order derivatives of Eq.(7), the instantaneous stiffness $k(t)$ and damping $c(t)$ can be eventually calculated by the measured $x(t)$ and its first two order derivatives. It means that this identification method with HT is a non-parameter method [6,7], i.e. only depends on outputs rather than the specific type of the nonlinearity. Thus, the nonlinear restoring and damping force characteristics can be approximately estimated by following symmetrical formulations

$$F_k(x)\approx\begin{cases}mk(t)A(t), & x>0\\ -mk(t)A(t), & x<0\end{cases},\quad F_c(\dot{x})\approx\begin{cases}mc(t)A_{\dot{x}}(t), & \dot{x}>0\\ -mc(t)A_{\dot{x}}(t), & \dot{x}<0\end{cases} \tag{10}$$

where $A_{\dot{x}}(t)$ is the envelope of the velocity.

3. Experimental identification

3.1 Experimental setup

The structure and components of the experimental nonlinear isolator is shown in Fig.2. This isolator mainly composes of four ring springs, in which one side of each ring is fixed on the rigid frame and the other side is fixed on mass block. The mass block is bolted to a guiding rod with the circular cross-section. Besides, the guiding rod is restrained by the two linear bearings fixed on the frame to guarantee that the isolator performs as a SDOF system. However, the existence of guiding rod and linear bearings introduces the dry friction, which is inevitable due to the gravity of the mass block. In addition, a reflection platform is fixed on the end of the rod for reflecting laser. Therefore, the equivalent mass of this isolator contains mass block, guiding rod, reflecting platform and the bolt connection parts related to the mass block. The measured total mass of these parts is 0.575kg.

Since the maximal deformation of the rings structure is up to10mm, a laser vibrometer system is used to measuring the nonlinear vibration. The Schematic diagram of the whole experimental system is demonstrated in Fig.3. The photograph in Fig.4 clearly shows the working condition of the isolator and the laser vibrometer. In this experiment, to avoid the asymmetry of nonlinear stiffness caused by the gravity, the isolator is mounted on the fixed base horizontally via a triangular support. The laser vibrometer is fixed right forward the isolator horizontally to ensure that the laser beam can focus on the central position of the reflection platform. Then, the measured data can be obtained and processed via the data acquisition system and PC. The sampling frequency is 500Hz and the concerned rang of frequency is 0~100Hz. The initial condition of free vibration is the obvious and steady initial deformation, which is generated by cutting off a hanging weight. Therefore, time responses of absolute displacement with different initial deformations can be obtained.

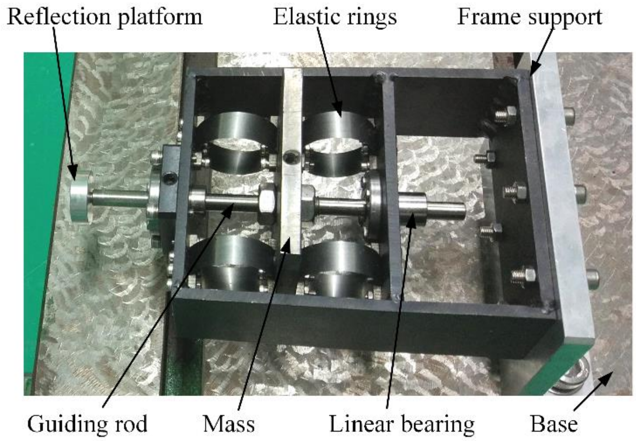

Fig. 2 Details of experimental nonlinear isolator

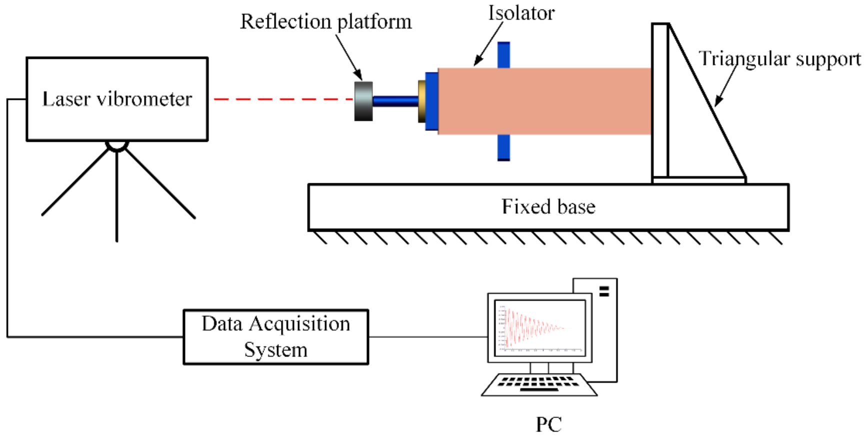

Fig. 3 Schematic diagram of the whole experimental system

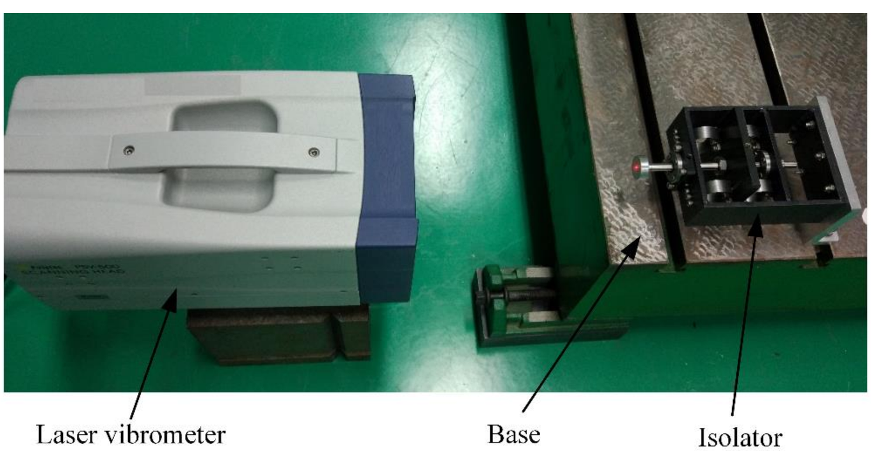

Fig. 4 Photograph of the experimental installation

3.2 Results and discussions

For obtaining more instantaneous characteristics, a group of measured displacement data with the maximal amplitude 12.98mm is used to identify the geometrically nonlinear isolator. Before using the Hilbert transform to the measured data, EMD is employed to decompose the original signal into several IMFs. Each decomposed IMF is actually a combination of some narrow frequency components. Only those IMFs which are adjacent to the system's range of the intrinsic frequency are eventually chosen to be the effective data for the Hilbert transform. It ensures that corresponding instantaneous modal parameters will have more specific physical significance.

Fig.5 shows the measured displacement with the EMD and its upper envelope formulated by Eq.(7). The instantaneous modal frequency can be calculated based on the identified stiffness coefficient (formulated in Eq.(9)) and is plotted in Fig.6. It shows that, with the vibration attenuation, the inherent frequency of the isolator decreases from more than 11Hz to 8.75Hz. It also indirectly demonstrates that the isolator with push-pull configuration rings performs like a hardening spring as the deformation increases.

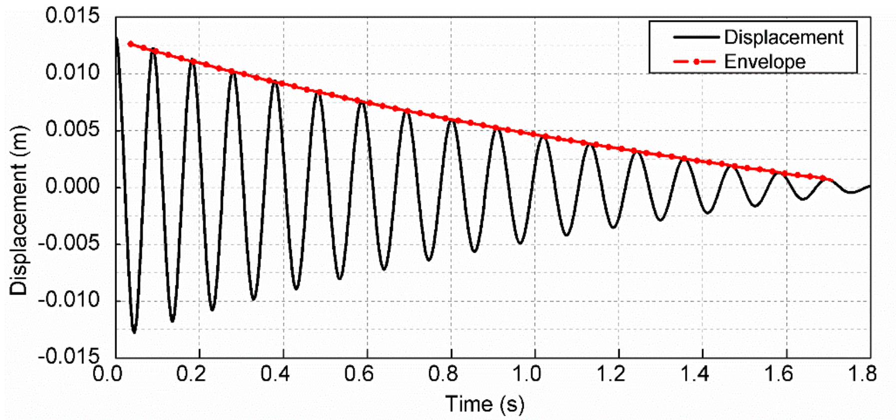

Fig. 5 The displacement and envelope of free vibration

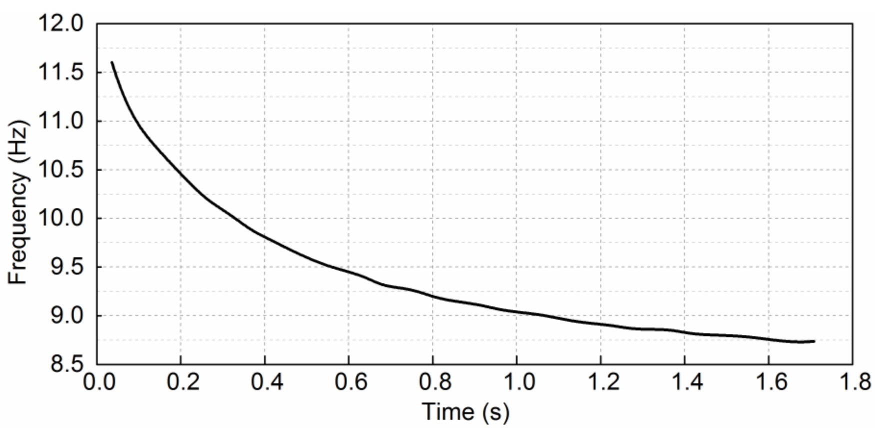

Fig. 6 The instantaneous modal frequency

After obtaining the envelope $A(t)$ and the instantaneous frequency $\dot{\psi}(t)$, the backbone curve of the isolator system can be plotted in Fig.7(a). A backbone curve directly depicts the relationship between the modal frequency and the amplitude of the system's response. In Fig.7(a), the identified backbone curve clearly demonstrates the hardening dynamic characteristics of the isolator with the push-pull configuration rings. To verify its validity and show it's physical significance, three groups of frequency responses under 0.2mm, 0.3mm and 0.4mm amplitudes of the sinusoidal base excitation are plotted together in Fig.7(a). Unstable solutions in each frequency response curve are identified based on the stability analysis theory proposed in Ref. [12] and are plotted in 'dash-line'. It should be known that these frequency responses are calculated based on the proposed analytical method and corresponding system identified by the frequency-domain experiment in the previous research [5]. Likely, other calculated results in following comparisons are all stemmed from this identified model. Fig.7(a) clearly shows that the identified backbone curve accurately passes through resonance peaks of the frequency response curve in Group i~ii and is fairly close to the one in Group iii. Thus, the identified backbone curve agrees well with the calculated frequency response based on the previously identified model.

Similarly, after obtaining the instantaneous damping coefficient $c(t)$, the damping curve which draws a modal damping coefficient as a function of the amplitude can be plotted in Fig.7(b). It is found that the damping coefficient of the isolator is dependent on the amplitude. It means that some nonlinear damping exists in the system and it may contain the dry friction. Specific forms of nonlinear damping will be noticeable after the system's damping force is identified.

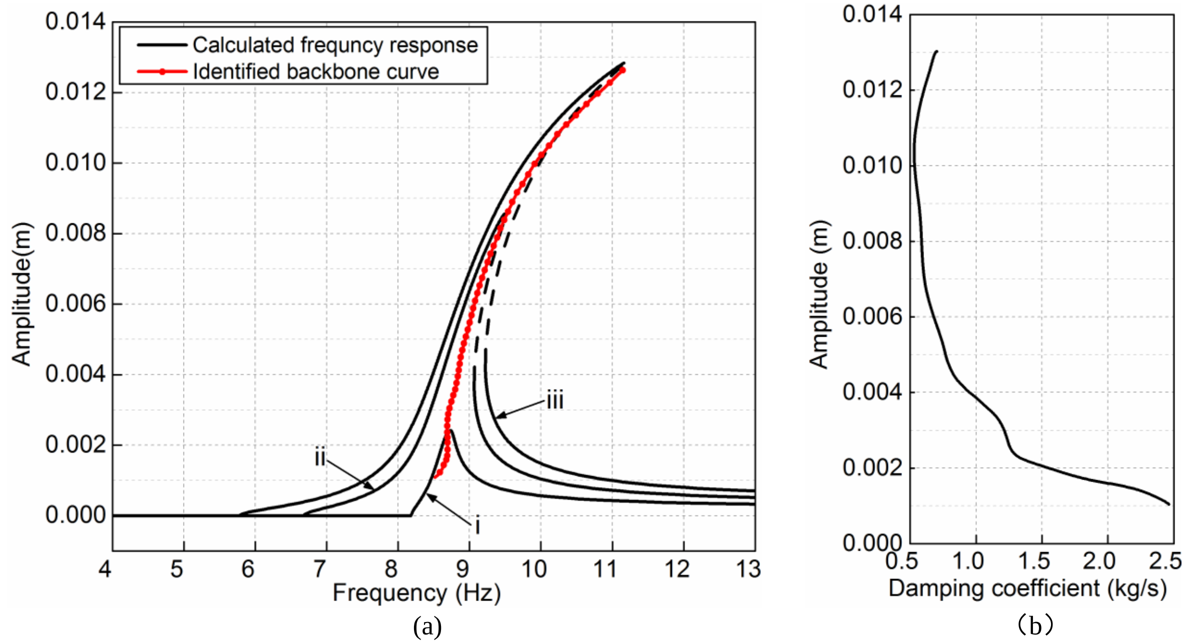

Fig. 7 (a). The identified Backbone curve and calculated frequency response under different amplitudes of sinusoidal base excitation: (i).0.2mm; (ii). 0.3mm; (iii).0.4mm; (b). The identified damping curve

For all the identification of a nonlinear isolator, the most significant and concerned part is its force static characteristics, including the restoring force and damping force. In our previous research [5], the nonlinear restoring force is obtained by both the numerical calculation based on the curved beam theory and the static experiment with an electro-mechanical testing machine. As for the nonlinear damping force, a combination of the viscous, coulomb and quadratic damping is verified by the frequency-domain vibration test.

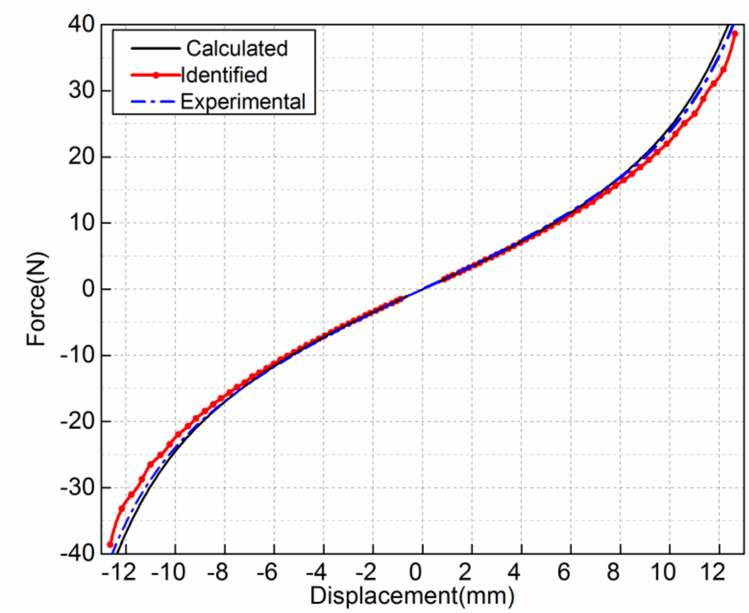

Fig. 8 Comparison between identified, calculated and experimental characteristics of the nonlinear restoring force

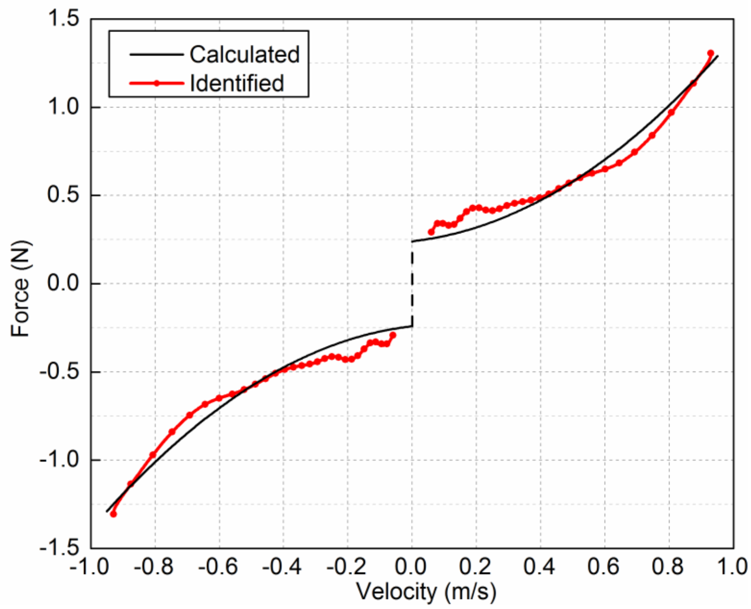

Fig. 9 Comparison between identified and calculated characteristics of the nonlinear damping force

Table 1 Correlation coefficients for evaluating identified results

Correlation coefficients	with calculated results	with static experimental results
Nonlinear restoring force	0.895	0.925
Nonlinear damping force	0.891	-

In this study, due to the identified instantaneous stiffness and damping coefficients, the nonlinear restoring and damping force can be approximately estimated by the maximum of displacement and velocity, respectively (formulated in Eq. (10)). Fig.8 and Fig.9 show comparisons between the identified results with the Hilbert transform and calculated/experimental results based on previous research. In Fig.8, identified restoring force is entirely identical with the calculated one when the displacement is no more than 6mm. However, some errors appear in the lager range of deformation on rings' structure and it may be caused by the elimination of high order oscillating components. In Fig.9, without any assumptions on the form of nonlinear damping, identified results by using the Hilbert transform well conforms to the previous research in the frequency-domain. The discontinuity part on the curve plotted in 'dash-line' denotes the effect of dry friction. The identified coefficient of coulomb damping is very closely to 0.241N, which is determined by the measured break-loose frequency in the frequency-domain experiment. The coherence degree between the identified result and the reference target can be evaluated by the correlation coefficient defined by

$$R = 1 - \sqrt{\sum \left(F_* - F_I\right)^2 \Big/ \sum F_*^2} \tag{11}$$

where F_* and F_I respectively denote the reference result and the identified result. A spline interpolation is used to ensure the consistency of independent variables. Table 1 lists correlation coefficients for evaluating identified results of restoring and damping force. In general, identified results of both the nonlinear restoring force and the nonlinear damping force agree well with the theoretically calculation and experimentally identification in the frequency-domain.

4. Conclusion

This paper applies the Hilbert transform to identify the dynamic characteristics of a geometrically nonlinear isolator with the push-pull configuration rings. An experimental system with a laser vibrometer is constructed to acquire time-domain responses with large initial deformations. The empirical mode decomposition is employed to preprocess the measured data and smooth all instantaneous modal parameters. The identified instantaneous modal frequency clearly demonstrates the variation range of the inherent frequency. The backbone curves, nonlinear restoring and damping force characteristics of this geometrically nonlinear isolator are identified based on time-domain experimental results. All identified results are compared with the previous frequency-domain test and good coherence validates the identification by using the Hilbert transform. Therefore, the Hilbert transform will be a convenient approach to analyze dynamic characteristics and design parameters for the geometrically nonlinear isolator with elastic rings.

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